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EFFICIENT ANALYTICAL APPROACH FOR NONLINEAR SYSTEM OF ADVANCED LORENZ MODEL

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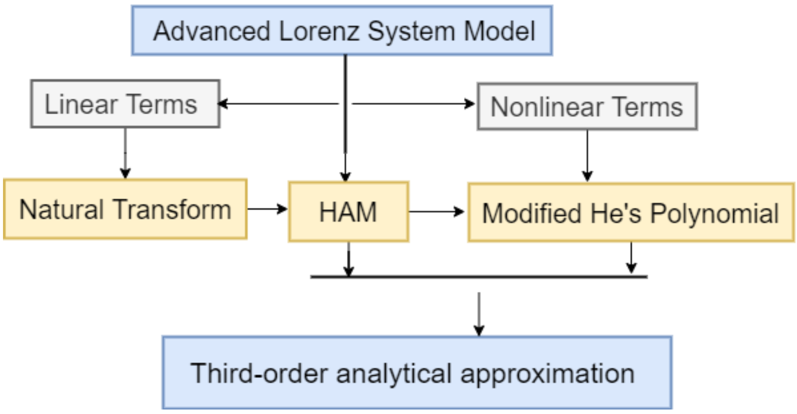
Research Highlights

This work proposed a new analytical approach for solving a famous model from mathematical physics, namely, advanced Lorenz system. The method combines the Natural transform and Homotopy analysis method, and it's have been suggested for the solution of various kinds for the systems of nonlinear delay differntial equations (DDEs). This technique generates solution in a polynomial series, where the modification of He's polynomial is successfully derived for the computions of nonlinear functions of the Lorenz system. By choosing an optimal value of auxiliary parameters the more precise approximation of the model is achieved at a maximum of three iterational number of terms. Some figures are used to demonstrate the accuracy of the result based on the residual error function. Therefore, the approach gives rise to an easy and straightforward means of solving these models analytically. Hence, it can be used in finding solutions to other forms of nonlinear problems.

Keywords:- Natural Transform, HomotopyAnalysis Method, He's Polynomial, Advanced, Lorenz System.

Graphical Abstract



Research Objectives

Delay differential equations rise in numerous fields in applied sciences and is very significant in the mathematical modelling of problems from real-life phenomena. Several problems from various fields of studies contain a delay element. The few of such include the biological species living together [1], the dynamics model of prey-predator which gives rise to delayed Volterra integro-differential equations [2] and the problems in population dynamics lead to the formation of delay logistic equation [3]. Partly due to the nature of the infinite-dimensional state they possess, methods of solving ODEs are not generally applied to DDEs. Therefore, the

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analytical solutions of these models are hardly ever available. Hence, they are mostly solved by numerical methods. In this research, an efficient analytical approach is applied to solve a model from mathematical physics namely the Advanced Lorenz system. This approach is developed in [4] for an efficient analytical approximation of different forms of the systems of nonlinear retarded delay differential equations (RDDEs). Therefore, the aim here is to find a better approximation analytically of this model using the proposed technique.

Methodology

The present work focus to obtain a better analytical approximation for Advanced Lorenz system model by using the approach developed in [4] for the systems of nonlinear RDDEs. This technique is from the combinational form of Natural transform (NT) and Homotopy analysis method (HAM) where the modification of He's polynomial is successfully derived for the computions of nonlinear functions. This technique provides solutions to various forms of nonlinear systems of RDDEs in a polynomial series that converged rapidly to an exact or approximate solutions using a maximum of three iteratioal numbers of terms. The idea here is use the Natural transform as a linear operator which is applied to obtain the simplified form of the linear term of such model where the concept HAM is used to construct the generating function for the computation of iterational terms. Lastly, the modified He's polynomial is defined for the calculating the nonlinear functions in the generating function constructed.

Results

Advanced Lorenz model is introduced by Xiao-hong et al. [5] and it is an advancement of the model proposed by Lorenz in [6] for atmospheric convection. So, according to Ansari and Dasi [7] the system of this model can be obtained as follows.

$$
\begin{aligned}
x_1'(t) &= 20[x_2(t) - x_1(t)] + 3x_1(t)(t-\tau) \\
x_2'(t) &= 14x_1(t) + \tfrac{53}{5}x_2(t) - x_1(t)x_3(t) \qquad (1) \\
x_3'(t) &= x_1^2(t) - \frac{14}{5}x_3(t),
\end{aligned}
$$

with initial condition

$$x_1(0) = -20, \quad x_2(0) = 8, \quad x_3(0) = 20$$

To find the approximation for the system in Eq. (1) based on the proposed technique, The NT of Equation should be first taking, and then by making the further simplification using the given initial condition the simplified form of the model is obtained as

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$$\mathbb{N}^+[x_1(t)] + \frac{20}{s} + \frac{20}{s}[x_1(t) - x_2(t)] - \frac{3u}{3}\mathbb{N}^+[x_1(t-\tau)] = 0$$

$$\mathbb{N}^+[x_2(t)] - \frac{8}{s} - \frac{14u}{s}\mathbb{N}^+[x_1(t)] - \frac{53}{5s}\mathbb{N}^+[x_2(t)] + \frac{u}{s}\mathbb{N}^+[x_1(t)x_2(t)] = 0 \qquad (2)$$

$$\mathbb{N}^+[x_3(t)] - \frac{20}{s} + \frac{14u}{5s}\mathbb{N}^+[x_3(t)] - \frac{u}{s}\ \mathbb{N}^+[x_1^2(t)] = 0$$

So, by following the procedure in [4] the recursive relation of the model can be obtained as

$$x_{1,m}(t) = (\chi_m + h_1)x_{1,m-1}(t) + h_1(1-\chi_m)\mathbb{N}^-\left[\frac{20}{s}\right] + h_1\mathbb{N}^-\left\{\frac{u}{s}\mathbb{N}^+\left[R_1\left(x_{1,m-1}, x_{2,m-1}\right)\right]\right\}$$

$$x_{2,m}(t) = (\chi_m + h_2)x_{2,m-1}(t) - h_2(1-\chi_m)\mathbb{N}^-\left[\frac{8}{s}\right] - h_2\mathbb{N}^-\left\{\frac{u}{s}\mathbb{N}^+\left[R_2\left(x_{1,m-1}, x_{2,m-1}\right) + H_{2,m-1}\left(x_{1,1}x_{3,1}, .. x_{1,N}x_{3,N}\right)\right]\right\} \qquad (3)$$

$$x_{3,m}(t) = (\chi_m + h_3)x_{3,m-1}(t) - h_3(1-\chi_m)\mathbb{N}^-\left[\frac{20}{s}\right] + h_3\mathbb{N}^-\left\{\frac{u}{s}\mathbb{N}^+\left[R_3\left(x_{3,m-1}(t)\right) + H_{3,m-1}\left(x_{1,1}(t), x_{1,2}(t), .. x_{1,N}(t)\right)\right]\right\} \quad m \geq 1,$$

where the nonlinear functions $x_1(t)x_2(t)$ and $x_1^2(t)$ are respectively calculated as the series of modified He's polynomials defined as

$$H_{2,m}\left(x_{1,1}x_{3,1}, .. x_{1,N}x_{3,N}\right) = \frac{1}{m!}\frac{\partial^m}{\partial q^m}F_2\left(\sum_{p=0}^{m} q^p\left(x_{1,p}(t), x_{2,p}(t)\right)\right)$$

and

$$H_{3,m}\left(x_{1,1}(t), x_{1,2}(t), .. x_{1,N}(t)\right) = \frac{1}{m!}\frac{\partial^m}{\partial q^m}F_3\left(\sum_{p=0}^{m} x_{1,p}(t)q^p\right).$$

Findings

From the given intial condition the intial approximation can be chosen as

$$x_{1,0}(t) = -20,\ \ x_{2,0}(t) = 8 \text{ and } x_{3,0}(t) = 20$$

So, from Eq. (3) at $h_1 = -0.997$ and $h_2 = h_3 = -1$, the third-order approximate solution of this model is given as follows:-

$$x_1(t) \approx \sum_{m=0}^{3} x_{1,m} = 32647.5t^3 - 2182.643t^2 + 498.4977t - 20$$

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$$x_2(t) \approx \sum_{m=0}^{3} x_{2,m} = -112116.777t^3 + 3029.94t^2 + 204.8t + 8$$

$$x_3(t) \approx \sum_{m=0}^{3} x_{3,m} = 122476.17t^3 - 1045.6t^2 + 498.4977t + 20$$

Therefore, based on the result obtained the most important thing about the proposed method is to make a good choose of the initial approximation, and this will accelerate the convergent for the series solutions and also determine the best set of function to be used, and to have better approximation from few numbers of iterations.

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Author's Biography

Aminu Barde *works with Abubakar Tafawa Balewa University, Bauchi as a lecturer from 2009 to date. He received his BSc Mathematics from Bayero University, Kano in 2006 and obtained his MSc in 2014 from Abubakar Tafawa Balewa University, Bauchi. He is currently a PhD student at Universiti Teknologi Malaysia. Aminu Barde has conducted various research in applied mathematics and the current research area is delay differential equations.*

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