A TWO-STRAIN DETERMINISTIC DENGUE MODEL WITH SEASONALITY EFFECT

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Author's Biography

Afeez Abidemi obtained his master's degree from the Federal University of Technology, Akure, Nigeria in 2015. He is currently a Ph.D student under the supervision of Dr. Mohd Ismail Abd Aziz and Dr. Rohanin Ahmad. His research is focused on mathematical modelling and optimal control of vector-borne diseases.

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RESEARCH HIGHLIGHTS

- A two-strain deterministic model of dengue outbreak in Madeira Island, Portugal is proposed.
- A modified two-strain dengue model which incorporates seasonal variation is presented.
- Model formulation and analyses of its basic properties are presented.
- Numerical simulation is carried out and the results are presented.

Basic Reproduction Number, Two-strain Dengue Model, Equilibrium Point, Stability, Dengue, Seasonality.

GRAPHICAL ABSTRACT

RESEARCH OBJECTIVES

The aim of this research is to formulate and analyse a deterministic model to examine the impact of seasonal variation on the dynamics of human population with primary and secondary infections and infected mosquitoes with serotypes 1 and j ($j \in 2,3,4$). In particular, we consider Madeira Island as a case study and use data from the first outbreak in 2012 as reported in (1) to predict dengue risk to the population when a second disease outbreak occur in the Island. There is the need for this study because most of the dengue-endemic regions have strong seasonal patterns in climatic factors like temperature and rainfall, which are directly linked to dengue disease transmission through the mosquito population (2).

MATERIALS AND METHODS

A compartmental model, which is based on the model presented in (1) and (2) is proposed to describe the dynamics of vector-host interactions for dengue disease transmission with

coexistence of two virus serotypes. The model is modified to incorporate seasonal-dependent mosquito birth rate in order to examine the influence of climatic factors such as rainfall and temperature on the dynamics of mosquito population and dengue disease transmission. The Next Generation Matrix method (3) is used to obtain the basic reproduction number, $\mathcal{R}_0 = \max\{\sqrt{\mathcal{R}_{01}}, \sqrt{\mathcal{R}_{0j}}\}$, associated with the model without seasonality effect. The global dynamics of the model is analysed using the Comparison Theorem (4). The model is simulated in MATLAB with ode45 routine for two cases, namely: the less aggressive case (Case 1) and the more aggressive case (Case 2).

RESULTS

Analysis of the model shows that the Disease-Free Equilibrium (DFE) is locally asymptotically stable whenever both the basic reproduction numbers $\mathcal{R}_{01}$ (associated with strain 1 only) and $\mathcal{R}_{0j}$ (associated with strain $j, j \in \{2,3,4\}$ only) are below unity. It is shown that the DFE is globally asymptotically stable when the susceptibility indices for secondary infection in strain 1 (σ_1) and strain j (σ_j), and $\mathcal{R}_{01}, \mathcal{R}_{0j}$ are all less than 1.

FINDINGS

The basic reproduction numbers for the outbreak ($\mathcal{R}_0 = \max\{2.0889, 1.7671\} = 2.0889$ for Case 1 and $\mathcal{R}_0 = \max\{2.0908, 3.1292\} = 3.1292$ for Case 2) are found to be above unity. In other words, DENV-1 is predominant in Case 1 while DENV-2/3 is predominant in Case 2. The basic reproduction values show that the disease will invade the population if a second outbreak occurs in Madeira Island. This is in agreement with other studies of dengue (5). It is found that the presence of seasonal pattern significantly increases the number of infected individuals and mosquitoes in both cases, which leads to higher expected number of disease-induced deaths.

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